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Computer-Aided Design using New Algorithms for nth Order Chebyshev Digital Filter

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Abstract This paper describes new algorithms for constructing the transfer function of the nth order Chebyshev digital LPF using the cascade combination of second order and first order digital filters. Using computer-aided design with new algorithms for collecting z-domain transfer function of (conjugate poles and single real pole) can be achieved. Linear programing for performing the collection of cascade combination is used to create C++ program for above requirements. The algorithms are very fast, flexible , and exact. Users of the program can achieve design of different sampling frequency, cut-off frequency, magnitude of passband ripple , and order of Chebyshev digital LPF with very high flexibility.

Keywords: Programming algorithms, Digital filter, IIR filter.

1. Introduction

This paper focuses on algorithms for design of Chebyshev digital LPF. Using the algorithms of this work it is very easy and flexible to achieve any order of Chebyshev digital filter which can be used for printed digital circuits or integrated circuits. The design is very fast which makes it very suitable to be used inside adaptive digital systems or real time digital systems.

The detection of the desired signals will be impossible if the undesired signals and noise are not removed by filtering. Electronic filter allows some signals to pass, but stops others. In another word electronic filter allows some signal frequencies applied at its input terminals to pass through to its output terminal with little or no reduction in signal level. Chebyshev filters have a sharp transition from the pass-band to the stop-band, which makes them very desirable for filtering[1].

Digital filtering is one of the most powerful tools of DSP. Digital filtering can be defined as the multiplication of the signal spectrum by the frequency domain impulse response of the digital filter[2].

Digital filters are desired in adaptive systems because their operation is determined by a program stored in the processor's memory. Which means digital filters can be easily changed without changing the hardware[3].

Chen [2] has gave a lot of information in digital filtering with additions in the area of computer-aided design of digital filters.

Thede [3] has written programs in C Language for IIR filter design. He has used bilinear transformation to convert transfer functions from s-domain to z-domain.

Chauhan[4] has provided algorithms for the design of IIR digital filter.

Kumar[5] describes CAD techniques for FIR and IIR digital filters.

2 Lowpass Chebyshev Filters

Chebyshev filters are designed to have an amplitude response characteristic that has a relatively sharp transition from the pass-band to stop-band . This sharpness is accomplished at the expense of ripples that are introduced into the response[2].

The Chebyshev approximation for ideal lowpass filter shows a magnitude that has the same values for the maxima and for the minima in the passband and decreases monotonically as the frequency increases above the cut-off frequency[3].

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The normalized magnitude of prototype lowpass filter is [5,6]:

$$\left|H(jw/w_c)\right| = \frac{1}{\sqrt{1+\varepsilon^2 \cdot C_n^2(w/w_c)}} \cdots\cdots\cdots\cdots\cdots (1)$$

Where Cn is nth order Chebyshev polynomial.

n is the order of the filter.

$$\varepsilon = \sqrt{10^{-0.1.A_p} - 1} \cdots\cdots\cdots\cdots\cdots (2)$$

Ap is the magnitude of passband ripple.

The transfer function in s-domain can be given [6,7]as:

$$H(s) = \frac{H_o}{\prod_{k=1}^{n}(s-p_k)} \cdots\cdots\cdots\cdots\cdots (3)$$

Where n is the order of the filter

Pk is the kth pole which can be calculated as shown[6,7]:

$$p_k = -\sin(\psi)\sin(\theta_k) + j\cos(\psi)\cos(\theta_k) \cdots\cdots\cdots\cdots (4)$$

Where

$$\theta_k = \frac{(2k-1)\pi}{2n} \cdots\cdots\cdots\cdots\cdots (5)$$

$$\psi = \frac{1}{n}\sinh^{-1}\left(\frac{1}{\varepsilon}\right) \cdots\cdots\cdots\cdots\cdots (6)$$

Therefore from equation (4) it is clear that all poles are complex conjugate except for odd n there will be single real pole.

3. Converting the Transfer Function from H(s) to H(z)

For the first order transfer function [7]:

$$H(s) = \frac{1}{s+a} \qquad (9)$$

$$\Rightarrow H(z) = \frac{1}{1 - e^{-aT}.z^{-1}} \qquad (10)$$

Where T is the sampling period.

The second order transfer function (with conjugate poles) [8]:

$$H(s) = \frac{1}{(s+\alpha+j\beta)(s+\alpha-j\beta)} \qquad (11)$$

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$$H(s) = \frac{j/2\beta}{s + \alpha + j\beta} - \frac{j/2\beta}{s + \alpha - j\beta} \qquad (12)$$

Now using the transformation of Equation (9) to Equation(10) in Equation(12),we get :

$$H(z) = \frac{j/2\beta}{1 - e^{-(\alpha+j\beta)T} z^{-1}} - \frac{j/2\beta}{1 - e^{-(\alpha-j\beta)T} z^{-1}} \qquad (13)$$

Then that leads to: $$H(z) = \frac{e^{-\alpha T} \sin(\beta T) z^{-1}/\beta}{1 - 2e^{-\alpha T} \cos(\beta T) z^{-1} + e^{-2\alpha T} z^{-2}} \qquad (14)$$

All the poles of Chebyshev filter are conjugate poles with single real pole for odd order transfer function as Equation (4) clarifies.

4. General Circuits and Equations

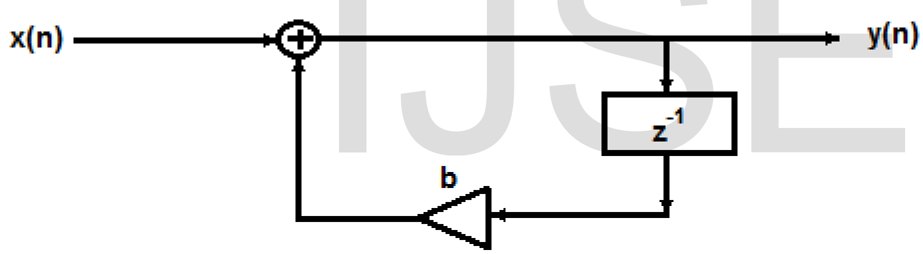
If we make a general IIR circuit for the following first order transfer function[3]:

$$H(s) = \frac{1}{s + a} \Rightarrow H(z) = \frac{1}{1 - e^{-aT}.z^{-1}} \qquad (15)$$

Now let $b = e^{-aT}$ (16)

Then $H(z) = \frac{1}{1 - b.z^{-1}}$ (17)

Then the IIR circuit for above Equation is:



.Figure (1): First order IIR circuit

Now the general IIR circuit for the following second order transfer function [3]:

$$H(z) = \frac{a1\, z^{-1}}{1 - b1\, z^{-1} - b2\, z^{-2}} \qquad (18)$$

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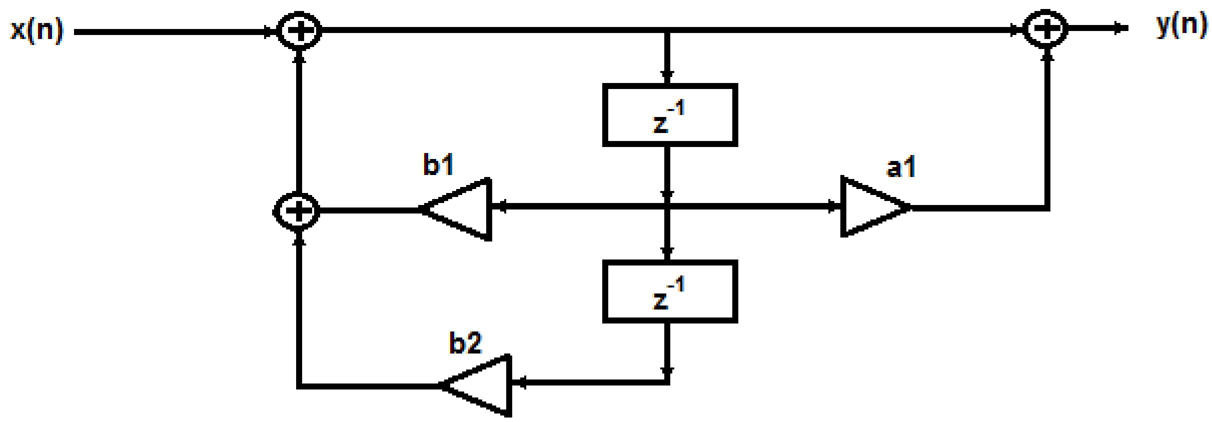

Figure (2): Second order IIR circuit for Equation (18).

Now by comparing Equation (14) with Equation (18), we get:

$$a1 = \frac{e^{-\alpha T}\sin(\beta T)}{\beta} \qquad (19)$$

$$b1 = 2e^{-\alpha T}\cos(\beta T) \qquad (20)$$

$$b2 = -e^{-2\alpha T} \qquad (21)$$

5. Design Principles

The nth order Chebyshev digital filter can be constructed using multistage . Each two conjugate poles give a stage of second order transfer function. When the order is odd then there is a real pole (s=-a) which can be given by a circuit like the circuit of Figure(1) with:

$$b = e^{-aT} = e^{-\sinh(\varphi)T} \qquad (22)$$

For example if the designed filter is 7th order then the stages will be as shown in Figure (3).

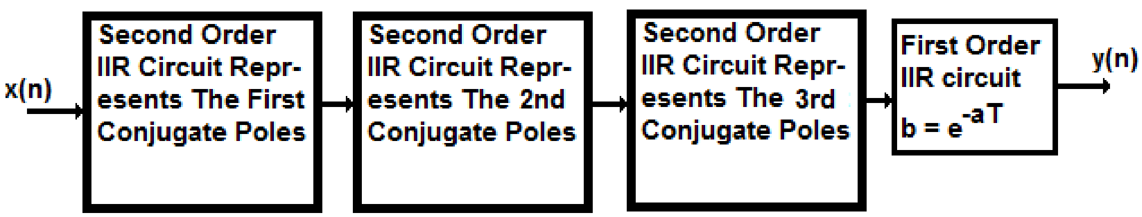

Figure (3): Stages of 7th order Chebyshev LPF.

The calculation of coefficients of each stage depends on the calculated conjugate poles by using equations (19), (20) , (21) and (22).

6. Realization of the Prototype Chebyshev Filter

The designed filter stands for prototype Chebyshev filter with cut-off frequency is 1 rad./sec., therefore the realization conversion must be done to convert to the real cut-off frequency . The realization procedure is by converting each s in H(s) to (s/wc). Where wc is the cut-off frequency in rad./sec. [3,6,9,10].

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7. Frequency Transformation

LPF has been discussed in this paper , but other types also can be represented easily by simple frequency transformation to get: (High-pass, Band-pass , or Band-stop) Filters [1 ,8].

8. Design Algorithms

Flowchart (1) shows the algorithms of nth order Chebyshev digital filter which gives high flexibility to choose (sampling, cut-off) frequencies, the magnitude of ripple of pass-band and the order of the Chebyshev digital filter.

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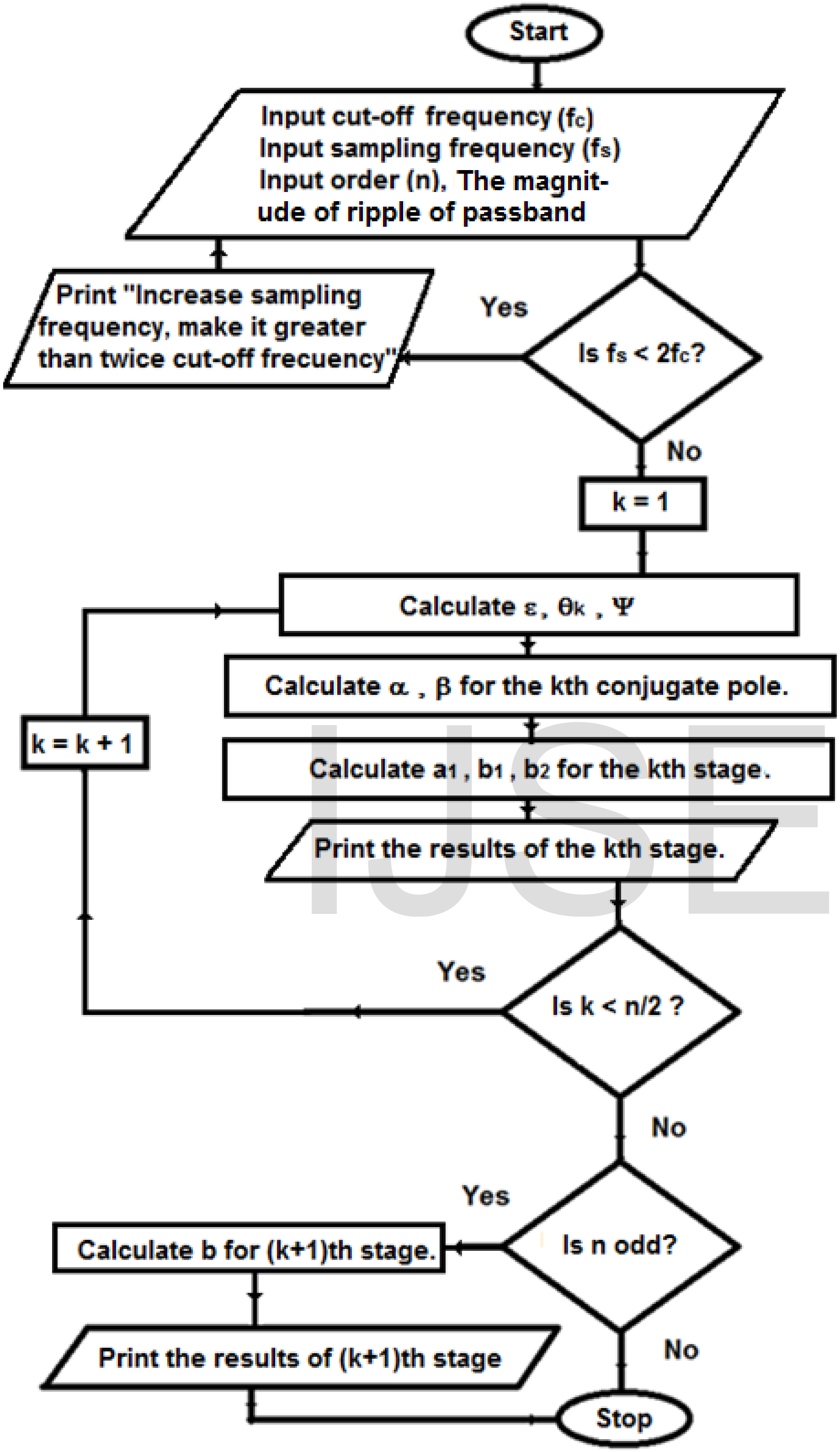

Flowchat(1): The algorithms of nth order Chebyshev digital filter design program.

9. DESIGN EXAMPLE RESULTS

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Example results of the program (in C++ Language) which follows the algorithms of Flowchart (1) are as follows:

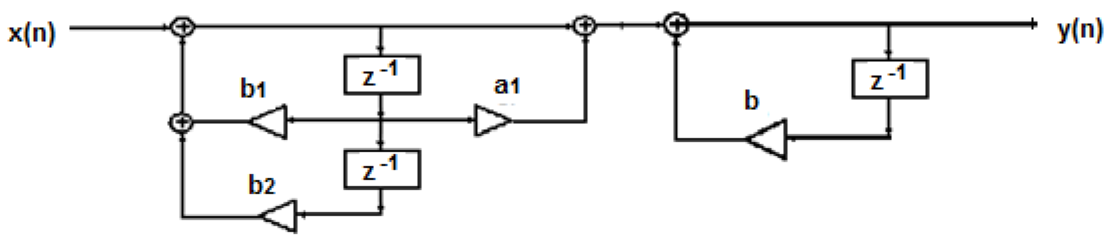
• For third order with sampling freq.=100kHz and with selected cut-off frequency fc=26kHz.

The magnitude of pass-band ripple Ap = 0.5 dB

• The first stage: b1=0.328598 , b2= -2.782631, a1=1.624392

• The second stage : b=1.668122

Fig.(4) shows above third order Chebyshev filter



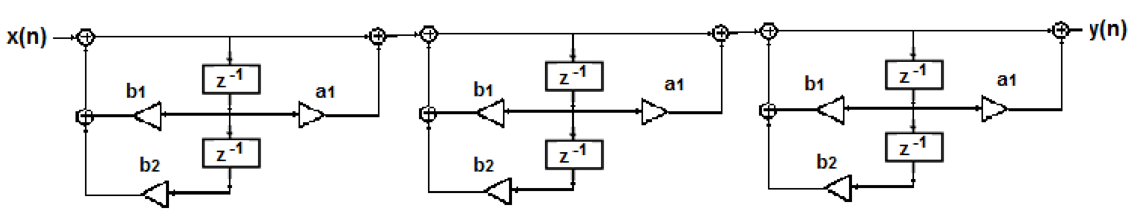
Figure(4): Third order Chebyshev LPF

For sixth order with sampling freq.=100kHz and with selected cut-off frequency fc=26kHz ,

The magnitude of pass-band ripple Ap = 0.5 dB

- The first stage: b1=-0.173872, b2=-1.288789 , a1= 1.122418
- The second stage: b1= 1.009012 , b2= -1.999963, a1= 1.789583
- The third stage: b1= 2.903138, b2=-2.577529 , a1= 2.538386

Fig.(5) shows above sixth order Chebyshev filter



Figure(5): Sixth order Chebyshev LPF

For tenth order with sampling freq.=100kHz and with selected cut-off frequency fc=26kHz , The magnitude of pass-band ripple Ap = 0.5 dB

- The first stage: b1=-0.142606 , b2=-1.095438 , a1= 1.040794
- The second stage: b1= 0.210305 , b2= -1.302833 , a1= 1.255779
- The third stage: b1=0.951169 , b2= -1.509883 , a1= 1.577422
- The fourth stage: b1=1.8911692, b2=-1.680666 , a1=1.923125
- The fifth stage: b1=2.577532 , b2= -1.778067 , a1= 2.153966

Fig.(6) shows above tenth order Chebyshev filter

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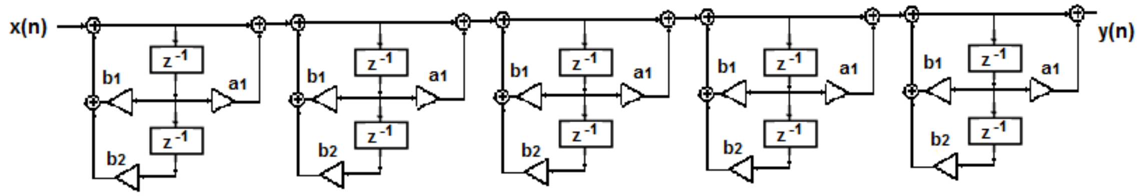

Figure(6): Tenth order Chebyshev LPF

10. CONCLUSIONS

The designed program for Chebyshev digital filter CAD, is very simple to be used and does not require knowledge in electronic filtering design in order to run it.

The performance of each designed filter has been evaluated using “Electronic Workbench 10” , which resulted that the design of program is exact and convenient for any particular application.

The main outputs from this program are the coefficients of each stage of the filter.

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